

Fluid Mechanics 101

Calculators & Tools



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1 Disclaimer

Before using these calculators, it should be emphasised that unverified sources of information on the internet should not be trusted or relied upon. The calculators themselves and equations provided in this document **have not been verified or checked** and are likely to contain errors. If you want to use the calculators or methods in this document yourself, you should check the original sources and equations themselves and implement them yourself in your own work. At which point, you must follow the appropriate verification and validation processes provided by your company, institution or establishment.

The calculators and equations provided here only provide an indication of how such a calculation could be carried out by an engineer. The equations and their implementation in the calculators should never be relied upon under any circumstances and are likely to contain errors.

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Never rely on unverified sources of information on the internet (this document in particular)! They are likely to contain errors.

2 Inflation Layer Calculator

First Cell Height

Start by calculating the flow Reynolds number (Re).

$$Re = \frac{\rho UL}{\mu} \quad (1)$$

where ρ is the fluid density, U is the freestream velocity, L is the characteristic length of the geometry and μ is the dynamic viscosity. An empirical correlation for fully developed turbulent flow over a flat plate is then used to estimate the skin friction coefficient (c_f):

$$c_f = [2 \log_{10}(Re) - 0.65]^{-2.3} \quad (2)$$

This particular correlation has been taken from Schlichting (1979) for fully turbulent flow with $Re < 10^9$. This correlation could be replaced with any other empirical correlation for turbulent flow over a flat plate. However, as this calculator is only used as an estimate, small differences arising from the choice of correlation do not matter significantly.

Having computed the skin friction coefficient, the wall shear stress (τ_w) is calculated.

$$\tau_w = \frac{1}{2} \rho U^2 c_f \quad (3)$$

The friction velocity (u_τ) can then be computed from the wall shear stress:

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}} \quad (4)$$

Finally, the equation for y^+ can be rearranged to give the height of the wall adjacent cell centroid from the wall (y_p):

$$y^+ = \frac{\rho y_p u_\tau}{\mu} \quad (5)$$

$$y_p = \frac{y^+ \mu}{u_\tau \rho} \quad (6)$$

The distance of the cell centroid from the wall is given by y_p while the height of the cell (y_H) is double y_p .

$$y_H = 2y_p \quad (7)$$

Most mesh generators give the user control over the node spacing normal to the wall. Hence, the overall size of the cell (y_H) is a more useful output from this calculator, as the size of the cell is equal to the node spacing. y_H can be used directly in the mesh generator.

Having created an initial mesh, an initial CFD analysis should be carried out with this mesh. The first cell height calculated with this calculator is **only an estimate** and will need to be updated using information from this initial CFD analysis. For example, if you were targeting $y^+ = 1$ and the initial CFD analysis showed $y^+ = 2.2$, then you would need to reduce your cell height by a factor of ~ 2.2 for your real calculations.

The first cell height calculated with this calculator is only an estimate because the equation for the skin friction coefficient is an empirical correlation for a specific flow scenario (turbulent flow over a flat plate without an applied pressure gradient). Real CFD calculations which use a different geometry are likely to have a similar skin friction coefficient to a flat plate, but not the same. Hence, the calculator can only be an estimate at best.

Growth Ratio

In addition to calculating the size of the first cell (y_H), most mesh generators require a growth ratio (r) to be specified normal to the wall. This allows the layers to inflate (grow) normal to the wall.

When constructing the mesh, it is conventional in CFD to aim for a number of cells through the thickness of the boundary layer. For aerodynamic flows using Reynolds-averaged Navier-Stokes (RANS) turbulence modelling, CFD engineers often aim for 5 - 30 inflation layers ($N = 5 - 30$) through the thickness of the boundary layer. We can use this criteria to calculate a maximum growth ratio normal to the wall.

Start by using empirical correlations to estimate the boundary layer thickness δ_{99} . If the flow is laminar, then the Blasius solution gives:

$$\delta_{99} = \frac{4.91L}{\sqrt{Re_L}} \quad Re < 5 \times 10^5 \quad (8)$$

Conversely, if the flow is turbulent then an alternative correlation is used:

$$\delta_{99} = \frac{0.38L}{Re_L^{1/5}} \quad Re > 5 \times 10^5 \quad (9)$$

This correlation has been taken from Cengel & Cimbala (2006, Section 10.6).

The next stage in the calculation is to calculate the total thickness of the inflation layers (y_T), for a given number of layers N and growth ratio r . We can then match this to the boundary layer thickness δ_{99} to allow the inflation layers to cover the boundary layer thickness.

The growth ratio r is the ratio between the height of two consecutive cells. As the cells get larger as we move away from the wall $r > 1$. We can write the total thickness of the first two inflation layers as:

$$y_T = y_H + y_H r \quad (10)$$

Using the same logic, the total thickness of the first three inflation layers is:

$$y_T = y_H + y_H r + y_H r^2 \quad (11)$$

It follows that the total thickness of N inflation layers is:

$$y_T = y_H + y_H r + y_H r^2 + y_H r^3 + \dots + y_H r^{N-1} \quad (12)$$

This can be written concisely with summation notation:

$$y_T = \sum_{k=0}^{N-1} y_H r^k \quad (13)$$

This is the formula of a *geometric series*. It is convenient to re-write the geometric series using the identity:

$$\sum_{k=0}^{N-1} y_H r^k = y_H \frac{1 - r^N}{1 - r} \quad (14)$$

You can find this identity on *Wikipedia* or in any mathematics textbook. Hence, the total height of the inflation layers is:

$$y_T = y_H \left(\frac{1 - r^N}{1 - r} \right) \quad (15)$$

We would like the total height of the inflation layers to be equal to (or greater than) the thickness of the boundary layer δ_{99} .

$$\delta_{99} = y_H \left(\frac{1 - r^N}{1 - r} \right) \quad (16)$$

It is difficult to solve this equation directly for the required growth ratio r . A numerical solution is more straightforward. Start by rearranging the equation.

$$r^N - r \left(\frac{\delta_{99}}{y_H} \right) + \left(\frac{\delta_{99}}{y_H} - 1 \right) = 0 \quad (17)$$

The growth ratio which results in an error of $f(r) = 0$, is the growth ratio that has the total height of the inflation layers equal the boundary layer thickness. Hence, we can use a root finding algorithm to solve the equation:

$$f(r) = 0 \quad (18)$$

In the authors experience, the Newton-Raphson iteration is an effective and reliable algorithm for solving this equation. Starting with an initial guess $r_0 = 5$ (found to be effective in the author's experience for this problem), the growth ratio can be updated with the following equation:

$$r_{i+1} = r_i - \frac{f(r_i)}{\frac{\partial f}{\partial r}(r_i)} \quad (19)$$

Where the function $f(r_i)$ and the gradient $\partial f / \partial r(r_i)$ are:

$$f(r_i) = r_i^N - r_i \left(\frac{\delta_{99}}{y_H} \right) + \left(\frac{\delta_{99}}{y_H} - 1 \right) \quad (20)$$

$$\frac{\partial f}{\partial r}(r_i) = N r_i^{N-1} - \frac{\delta_{99}}{y_H} \quad (21)$$

Once the root finding algorithm has converged, the solution (r) is the maximum growth ratio that will result in the inflation layers being equal to the boundary layer thickness. In reality, the user will normally specify a growth ratio less than this, to ensure that the boundary layer thickness is within the inflation layers.

Final Layer Thickness

When applying inflation layers, it is desirable that the transition from the final inflation layer to the freestream mesh does not lead to a significant change in cell volume. This is particularly important for Large Eddy Simulation (LES), as the sub-grid viscosity is proportional to the cell volume. Hence, sudden changes in cell volume result in sudden changes in sub-grid viscosity, which can lead to inaccuracies and instability.

To make an assessment of the change in cell volume from the inflation layers to the freestream mesh, a simple approach is to compare the thickness of the final inflation layer with the length scale of the freestream mesh. The final layer thickness is:

$$y_{\text{Final}} = y_H r^{N-1} \quad (22)$$

This calculation is carried out at the end, as it requires the growth ratio r (which was computed in the previous section).

Mesh Quality

It should be emphasised that **these criteria alone (first cell height, growth ratio, final layer height) are not sufficient** to determine if a mesh is suitable for CFD analysis. The user will also need to check the mesh quality metrics and overall level of resolution in regions of high gradient, to ensure that the mesh is sufficient. These criteria will vary on a case-by-case basis and should be considered carefully by the user.

References

1. H. Schlichting and K. Gersten *Boundary-layer Theory*, ISBN 0-07-055334-3, 7th Edition, 1979.
2. Y. Cengel and J. Cimbala *Fluid Mechanics : Fundamentals and Applications*, ISBN 0-07-247236-7, McGraw-Hill Education, 2006.
3. Wikipedia, *Geometric Series*, Accessed 2021.

3 Inlet Turbulence Conditions Calculator

Start by calculating the turbulent kinetic energy (k) from the turbulence intensity (I) and the inlet velocity (U):

$$k = \frac{3}{2} U^2 I^2 \quad (23)$$

Next the user needs to specify the length scale (l) of the turbulence at the inlet. For an internal pipe flow, this is usually $\sim 10\%$ of the pipe diameter. For external flows the length-scale is likely to be significantly different and an appropriate value should be chosen carefully, depending on the ambient conditions.

Using the turbulent length-scale (l), the turbulent dissipation rate (ϵ) is calculated next. $C_\mu = 0.09$ is a model coefficient of the $k - \epsilon$ and $k - \omega$ turbulence models.

$$\epsilon = C_\mu \frac{k^{3/2}}{l} \quad (24)$$

From the definition of the specific dissipation rate, ω is calculated next.

$$\omega = \frac{\epsilon}{C_\mu k} \quad (25)$$

Finally, the kinematic viscosity is computed:

$$\nu_t = \frac{k}{\omega} \quad (26)$$

4 Particle Settling Velocity Calculator

Start with an initial guess for the settling velocity of the particle (U), by taking Stokes solution for the terminal settling velocity of a spherical particle falling in a viscous fluid.

$$U = \frac{gD^2}{18\mu}(\rho_p - \rho_f) \quad (27)$$

where g is the acceleration due to gravity (9.81 m/s^2), D is the diameter of the particle, μ is the dynamic viscosity of the fluid, ρ_p is the density of the particle and ρ_f is the density of the fluid.

However, Stokes solution is only valid for $Re < 1$. Hence, evaluate the particle Reynolds number (Re) next:

$$Re = \frac{\rho UD}{\mu} \quad (28)$$

With the particle Reynolds number, we can evaluate the actual drag coefficient of the particle using the Schiller-Naumann empirical model.

$$C_D = \begin{cases} \frac{24}{Re} (1 + 0.15Re^{0.687}) & Re < 1000 \\ 0.44 & Re > 1000 \end{cases} \quad (29)$$

Now that the drag coefficient has been calculated, an updated guess can be made for the particle settling velocity. To simplify the iteration process, define a drag function f :

$$f = \frac{C_D Re}{24} \quad (30)$$

With the drag function computed, update the settling velocity using a modified form of Stokes solution:

$$U = \frac{gD^2}{18\mu f}(\rho_p - \rho_f) \quad (31)$$

Now continue to iterate, by calculating the particle Reynolds number, the drag function and the settling velocity until converged. Use under-relaxation for stability.

5 Humidity Calculator

Start by calculating the saturated vapour pressure (P_g) using the Arden-Buck equation. Note that the temperature (T) is in °C (not Kelvin) and the calculated vapour pressure is in kPa.

$$P_g = \begin{cases} 0.61121 \exp \left[\left(18.678 - \frac{T}{234.5} \right) * \left(\frac{T}{T+257.14} \right) \right] & T > 0^\circ\text{C} \\ 0.61121 \exp \left[\left(23.036 - \frac{T}{333.7} \right) * \left(\frac{T}{T+279.82} \right) \right] & T < 0^\circ\text{C} \end{cases} \quad (32)$$

Now calculate the partial pressure of the water vapour (P_{H20}) in the air-water vapour mixture, by rearranging the definition of the relative humidity (ϕ):

$$\phi = \frac{P_{H20}}{P_g} \quad (33)$$

$$P_{H20} = \phi P_g \quad (34)$$

Next, calculate the specific humidity (ω), which is defined as the ratio of the mass of water-vapour to the mass of dry air:

$$\omega = \frac{m_{H20}}{m_{air}} \quad (35)$$

$$\omega = \frac{M_{H20}}{M_{Air}} \left(\frac{P_{H20}}{P - P_{H20}} \right) \quad (36)$$

NOTE: This equation can be derived by applying the ideal gas law to the dry air and water vapour individually. (V) is the gas volume, (R_{Air}) is the specific gas constant for air, (R_U) is the universal gas constant and (M_{Air}) is the molar mass of air.

$$P_{Air} V = m_{Air} R_{Air} T \quad (37)$$

$$P_{H20} V = m_{H20} R_{H20} T \quad (38)$$

$$\omega = \frac{m_{H20}}{m_{Air}} \quad (39)$$

$$\omega = \frac{R_{Air} P_{H20}}{R_{H20} P_{Air}} \quad (40)$$

$$\omega = \frac{(R_U/M_{Air}) P_{H20}}{(R_U/M_{H20}) P_{Air}} \quad (41)$$

Apply Dalton's Law of partial pressures:

$$P = P_{Air} + P_{H20} \quad (42)$$

$$\omega = \frac{M_{H20}}{M_{Air}} \left(\frac{P_{H20}}{P - P_{H20}} \right) \quad (43)$$

Finally, compute the absolute humidity (φ) from the specific humidity. The absolute humidity is defined as the mass of water vapour per unit mass of air-water vapour mixture.

$$\varphi = \frac{m_{H20}}{m_{Air} + m_{H20}} \quad (44)$$

$$\varphi = \frac{m_{H20}/m_{Air}}{1 + m_{H20}/m_{Air}} \quad (45)$$

$$\varphi = \frac{\omega}{1 + \omega} \quad (46)$$

The mass-fraction of water vapour (Y_{H20}) in the CFD computations is equivalent to the absolute humidity. Apply this at the inlet to the CFD computations:

$$Y_{H20} = \varphi \quad (47)$$

References

1. Buck, A. L. (1981), 'New equations for computing vapor pressure and enhancement factor, J. Appl. Meteorol., 20: 1527–1532
2. Buck, A. L. (2012), Model CR-1A Hygrometer with autofill operating manual, Buck Research Instruments, LLC.